**Mathematical Methods in Earth Sciences**

Lecture 4 – Mar/26/2017

Dot and Cross Products

So far, we haven’t talked about vector multiplication. It turns out that there are two ways of doing it: (1) the *dot product*, which produces a scalar, and (2) the *cross product* which produces a vector.

*The Dot (or Scalar) Product*

**When is it useful?** Anywhere you need to find the component of something which acts in a particular direction.

Given two vectors and , we define the dot product of and as

where and . Note that the dot product is a *scalar*.

**Example:** If and then what is ?

The dot product is

1. Commutative
2. Distributive
3. Associate

Physically, we think of the dot product as the projection of one vector onto another. You can also think of the dot product measuring how parallel and are. Obviously, the angle between the two vectors has an important role.



A very important result is the following (which can be proved (how?) with the law of cosines):

cos



Remember that gives a scalar. The scalar is the length of times the length of the component of projected onto (or vice versa).

There are two special cases. If , then the two vectors are perpendicular and the dot product is 0 (cos). If , then the two vectors are parallel and the dot product is simply .

**Example** The work done is distance travelled times by force applied *in the direction of travel*. We can write this as

where and are the vectors describing the force and travel vectors, respectively.

The most useful thing about the dot product is that it gives us an easy way of *calculating the angle between two vectors in 3D*. Let’s say that we know the two vectors and . Then the angle between them is given by

cos

**Example** What’s the angle between [−1, 0, 1] and [−1, 1, 1]?

**Caution** The dot product *only* works with Cartesian vectors!

For *unit* vectors, the angle between the two vectors is given by

cos

This expression is particularly useful if we are dealing with geographic (lat, long) coordinates. For instance, given two points on a sphere () and (), you can show that the angle between them is given by

cos



*Cross (or Vector) Product*

**When is it useful?** It appears in the fundamental equations governing magnetic induction, torque and the Coriolis effect. Also useful when you want to define mutually orthogonal axes.

Given two vectors and , we define the cross product of and as

or equivalently

where

Note that the resulting answer is also a vector (unlike the dot product).

**There is a short-hand way of remembering this formula**

**Example:** If and then what is ?

The cross product has several useful properties:

1. is perpendicular to both and . The cross product is normal to the plane containing the two vectors. Thus .
2. obeys the right-hand rule. So if is oriented along and is oriented along , then will be oriented along . Another way of writing this is: .
3. . Thus if , then . In other words, parallel vectors have a cross product of the null vector (zero). So the cross product measures how orthogonal two vectors are.
4. is equal to the area of the parallelogram defined by and .

*If the* **cross product** *of two vectors is zero, the vectors are* **parallel***.*

*If the* **dot product** *of two vectors is zero, the vectors are* **perpendicular***.*

The cross product is

1. Anti-commutative **why?**
2. Distributive
3. *Not* associative

**Example** - Derive the following equations

The law of cosines *a*2=*b*2+*c*2-2*bc* cos*A* using dot product

The law of sinesusing cross product



**돌림힘(torque)**은 물체를 회전시키는 데 작용한 힘이다. 회전 중심점(pivot point)으로부터 힘의 작용점까지의 거리를 모멘트의 팔(moment arm)이라 한다.

** **

물체를 돌리기 위해서는 모멘트의 팔에 수직으로 작용해야 한다. 단위는 N로 이는 joule의 단위로 에너지에 해당하지만 에너지는 아니므로 항상 N로 기록하고 힘으로 해석해야 한다. 문을 열 때 경첩에 가까운 곳과 먼 곳에서 힘을 작용할 때의 토크를 비교하시오.



**각속도(angular velocity)**

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호의 길이: , 선속도:



*Triple Product*

There are two products involving three vectors, one called the *scalar triple product* (because the answer is a *scalar*) and the other called the *vector triple product* (because the answer is a *vector*).

*Vector triple product* is written A useful identity is that

Note that both sides of this identity are vectors.

We can combine the dot and cross products into the *scalar triple product*

which has the useful property of giving the volume of the parallelepiped defined by , and . If it turns out to be negative, it’s because the vectors are left-handed rather than right-handed.

각운동량(angular momentum):